# Use of Frequency Derivatives in the 3-D Full-Wave Spectral Domain Technique

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## *Abstract*

**Rational function approximations are used to extrapolate the frequency response of the scattering coefficients of 3-D structures. The rational functions are constructed by applying Padé approximation techniques to single frequency solutions of the currents and the derivatives of the currents with respect to frequency. The currents and current derivatives are computed using a modified spectral domain technique. The efficiency of the method, along with the direct determination of the poles and zeros of the transfer function, make the method well suited for Model-Based Parameter Estimation (MBPE). Multi-frequency-point Padé approximations are also investigated.**

## *Introduction*

In this paper electromagnetic simulations are used to generate pole-zero based passive microwave circuit models. The presented method determines the currents and the derivatives of the currents with respect to frequency using a modified spectral domain technique (SDT). The implementation of the method of moments using frequency derivative information has been demonstrated in [1-4] for free-space scattering problems. The significant contribution of this work is the incorporation of frequency derivative information into the SDT, to allow the extrapolation of the frequency response of a 3-D structure from information computed at one frequency point. A related technique is Asymptotic-Waveform-Evaluation (AWE) [5], where low order rational functions are used to approximate circuit responses. In references [1-5], efficient methods for determining the required derivatives are available, whereas in this work the functional form of the SDT Green's functions requires an analytically complex frequency derivative evaluation.

## *Basic Principles*

### Model-Based Parameter Estimation (MBPE)

MBPE is based on the premise that the desired N-port frequency response is well approximated in a limited bandwidth by a rational function of the form given in [4] as

$$S(f) \approx \frac{\sum_{i=0}^{M} a_i f^i}{\sum_{i=0}^{P} b_i f^i} \qquad (1)$$

The concept of analytic continuation allows expression of the desired frequency response as a Taylor series expanded about a fixed frequency $f_0$. A truncated Taylor series of the response is given by

$$S(f) \approx \sum_{i=0}^{N} c_i (f - f_0)^i \qquad (2)$$

where the coefficients $c_i$ are found directly from the frequency derivatives evaluated at $f_0$. The Padé approximation technique is a method for finding the coefficients of the rational function approximation in Equation 1 such that the derivatives up to order N of the rational function and the truncated Taylor series are equal at $f=f_0$ [6].

### Spectral Domain Method

A 3-D spectral-domain method using a rectangular PEC enclosure filled with piece-wise constant dielectric is used to solve for the currents on the conductors (similar to [7-9]). Rooftop basis functions are used for the $x$ and $y$ currents and rectangular cross-section current vias are used for the $z$-directed currents.

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**Determination of Current Distribution Derivatives**

An example entry for the moment matrix is given by

$$Z_{pq}^{xy} = \sum_{m} \tilde{J}_{ypm} \tilde{J}_{xqm} \tilde{G}'_{xy_{m,pq}} \quad (3)$$

which expresses the coupling between basis $p$ and $q$, where $m$ is a double index over the modes. The $n$th order derivative of the moment matrix entry with respect to frequency is given by

$$\frac{d^n Z_{pq}^{xy}}{df^n} = \sum_{m} \tilde{J}_{ypm} \tilde{J}_{xqm} \frac{d^n \tilde{G}'_{xy_{m,pq}}}{df^n} \quad (4)$$

Equation 4 is identical to the moment matrix entry, except the Green's function is replaced with its $n$th derivative. This allows the application of a database approach (similar to that used in [7,8] for moment matrix generation) for determining the moment matrix derivatives. The derivatives of the conductor currents are found (similar to [4]) from

$$\frac{d^n[J]}{df^n} = -[Z]^{-1} \sum_{i=0..n-1} \binom{n}{i} \frac{d^{n-i}[Z]}{df^{n-i}} \frac{d^i[J]}{df^i} \quad (5)$$

which gives a recursive method of determining the derivatives of the current from the derivatives of the moment matrix. In the examples, a maximum differentiation order of twelve is used.

**De-Embedding**

The $y$-parameters are determined by solving for the port currents generated from a gap voltage source at the edge of the enclosure. The S-parameters for the circuit are then computed directly from the $y$-parameters. The de-embedding is presented below.

The term "derivatives" is interpreted here as the derivatives of a variable with respect to frequency, evaluated at a particular frequency. The de-embedding procedure begins with the solution and its derivatives for the entire structure (including port discontinuities and feed lines). A differentiable version of the de-embedding procedure given in [8] is applied to this solution, resulting in the solution (and its derivatives) for the de-embedded structure. Any other port parameter (and its derivatives) can be found by application of standard differentiation rules to the port parameter transformations. A MBPE of the desired transfer function is then found from the derivatives of the de-embedded solution.

## Examples

**Meander Line**

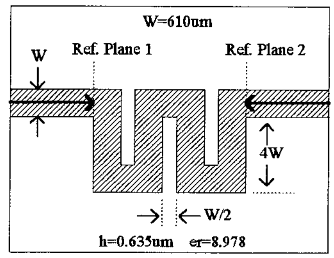


Figure 1. Meander Line Structure from [7]

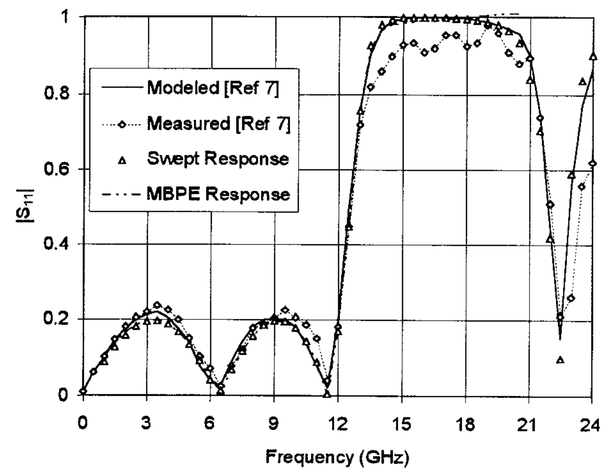


Figure 2. Magnitude of $S_{11}$ for Meander Line

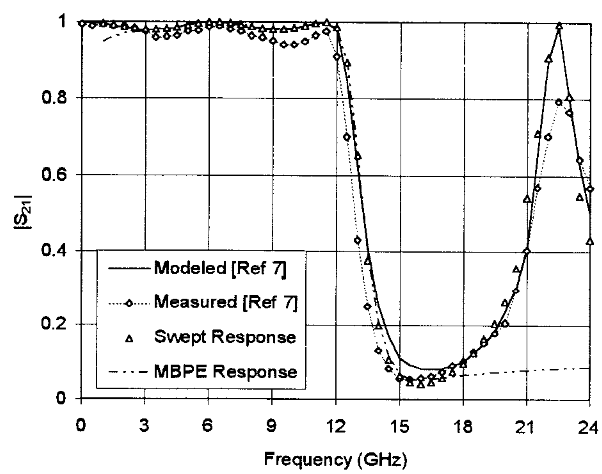


Figure 3. Magnitude of $S_{21}$ of Meander Line

A point-by-point frequency simulation of a meandering line is compared to a MBPE simulation of the same structure, computed from the derivatives evaluated at 9 GHz. The results, along with results and measured data from [7], are shown in Figures 2 and 3. The log of the error between the swept approach and the MBPE approach is shown in Figure 4 for the magnitude and angle of $S_{21}$.

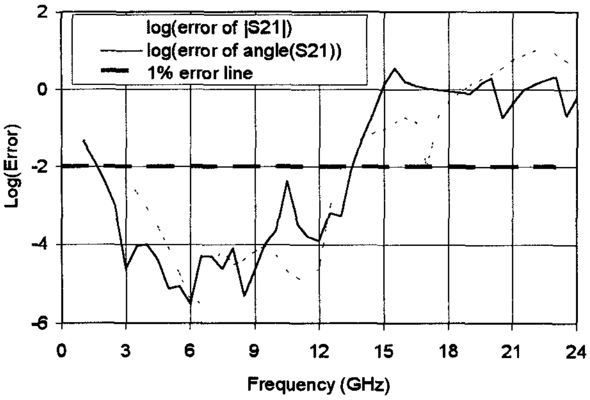


Figure 4. MBPE Error for Meander Line

**Spiral Inductor**

The spiral inductor shown in Figure 5 is analyzed and compared to simulated and measured results in [10]. A multi-frequency-point Padé approximation is used for the determination of the MBPE approximation of $S_{22}$.

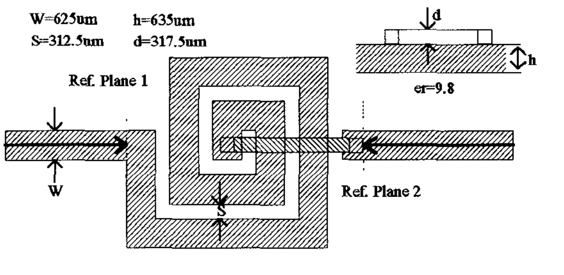


Figure 5. Spiral Inductor Example from [10]

The discrepancy between the computed data and the results from [10] can be attributed to the radiation losses from the open substrate simulation in [10]. The MBPE approximation was computed from the currents and their derivatives at 8 GHz and 16 Ghz. The difference between the MBPE solution and the point-by-point solution for $S_{22}$ is shown in Figure 7.

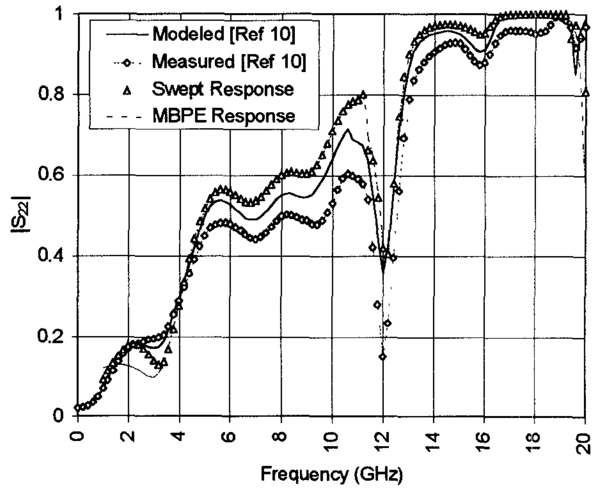


Figure 6. Magnitude of $S_{22}$ of Spiral Inductor

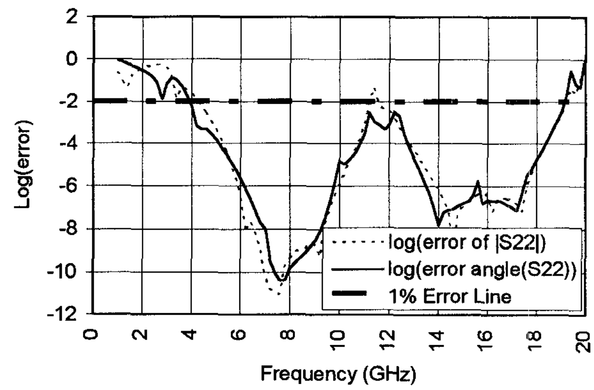


Figure 7. Multi-Point Padé Approximation Error

**Computation Times**

The times required to compute the solution at one frequency point and the times required to compute the expansion for the MBPE at one frequency point are given along with total time comparisons for the swept responses in the frequency ranges where the errors are within one percent. The time required to compute the derivative database is listed separately since this computation only needs to be performed once for a given substrate/enclosure configuration and expansion frequency. For both the meander line and spiral simulations, the MBPE approach is shown to be about an order of magnitude faster than the point-by-point approach. If a pre-computed derivative database is used, then the meander line computation time is 14 times faster, and the spiral computation time is 23 times faster.

| | *MBPE* | *Point-by-Point (2, 2.5 ... 13 GHz)* |
|---|---|---|
| Database Computation / freq. point | 5.1 min. | 0.2 min. |
| Current Solve Time/ freq. point | 13.7 min. | 8.2 min. |
| Total Solution Time / freq. point | 19 min. | 8.6 min. |
| Total Database Computation | 5.1 min. | 4.6 min. |
| Total Current Solve Time | 13.7 min. | 188.6 min. |
| Total Solution Time | 19 min. | 197.8 min. |

Table 1. Time Comparisons for Meander Line Analysis

| | *MBPE* | *Point-by-Point (4.6, 4.8 ... 19 GHz)* |
|---|---|---|
| Database Computation / freq. point | 35.6 min. | 0.73 min. |
| Current Solve Time/ freq. point | 25. min. | 15 min. |
| Total Solution Time / freq. point | 61 min. | 16 min. |
| Total Database Computation | 71.2 min. | 53.3 min. |
| Total Current Solve Time | 50 min. | 1095 min. |
| Total Solution Time | 122 min. | 1148.3 min. |

Table 2. Time Comparisons for Spiral Inductor Analysis

## Conclusion

The frequency responses of 3-D microwave structures are extrapolated from information at single frequency points, leading to significantly reduced computation times. The direct determination of the poles and zeros of the frequency response eliminates the need for closely spaced frequency points in a point-by-point swept frequency simulation. The poles and zeros can also be used to create a model of the structure. The ability to quickly generate models for many geometrical variations of a structure makes efficient passive structure model generation and optimization possible.

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